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Helicity Probabilities For Heavy Quark Fragmentation Into Heavy-light Excited Mesons *†

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ABSTRACT

After a brief review on how heavy quark symmetry constraints the helicity fragmentation probabilities for a heavy quark hadronizes into heavy-light hadrons, we present a heavy quark fragmentation model to extract the value for the Falk-Peskin probability $w_{3/2}$ describing the fragmentation of a heavy quark into a heavy-light meson whose light degrees of freedom have angular momentum $\frac{3}{2}$. We point out that this probability depends on the longitudinal momentum fraction z of the meson and on its transverse momentum $p_\perp$ relative to the jet axis. In this model, the light degrees of freedom prefer to have their angular momentum aligned transverse to, rather than along, the jet axis. Implications for the production of excited heavy mesons, like D^{**} and B^{**}, are briefly discussed.

Heavy quark Q produced at a high energy reaction can come with a very high degree of polarization. For example, at the Z resonance the charm and bottom quarks are predicted to be 67 and 94% left-handed polarized, respectively, in the Standard Model. An interesting question to ask is to what extend this large initial polarization of the heavy quark Q can be retained during the process of fragmentation. This hadronization process involves redistribution of energy at a smaller scale and thus nonperturbative QCD effects are overwhelmingly important. Since heavy quark spin and flavor are decouple from the dynamics at the limit $M_Q \to \infty$, heavy quark effective theory provides a powerful tool to analyze this problem. Indeed, Falk and Peskin [1] pointed out recently that heavy quark symmetry can impose very tight constraints on the helicity probabilities of heavy-light hadrons produced by the fragmentation/hadronization of a heavy quark.

Heavy quark symmetry implies heavy-light hadrons can be classified into $SU(2)$ doublets (H, H^*) with total angular momentum $(s, s^*) = (j_l - \frac{1}{2}, j_l + \frac{1}{2})$ respectively, where j_l is the eigenvalue of the angular momentum (J_l) of the light degrees of freedom. For instance, in the quark model picture of a heavy-light meson, one can write $J_l = S_{\bar{q}} + L$ where $S_{\bar{q}}$ is the spin of the antiquark and L is the orbital angular momentum. Let $P^H_{Q,h_Q \to s,h}$ denote the probability for a heavy quark Q with helicity h_Q (along the fragmentation axis) to fragment into a heavy-light hadron H, with angular

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momentum of the light degrees of freedom j_l, total angular momentum s, and total helicity h. Let $p_{j_l}(h_l)$ denote the probability for the heavy quark to fragment into the light degrees of freedom with angular momentum j_l and helicity h_l. Then $P^H_{Q,h_Q\to s,h}$ is given by a incoherent sum of $p_{j_l}(h_l)$ over all possible h_l weighted by the square of the Clebsch-Gordan coefficient $\langle s_Q, h_Q; j_l, h_l | s, h\rangle$ [1,2]

$$P^H_{Q,h_Q\to s,h} \propto \sum_{h_l} p_{j_l}(h_l) |\langle s_Q, h_Q; j_l, h_l | s, h\rangle|^2 \; . \tag{1}$$

An identical formula holds for H^* as well and will therefore be omitted. We will call $\{p_{j_l}(h_l)\}$ the Falk-Peskin probabilities. They are conditional probabilities and satisfy the following constraints: $(i)\, 0 \le p_{j_l}(h_l) \le 1$, $(ii)\, \sum_{h_l} p_{j_l}(h_l) = 1$, and $(iii)\, p_{j_l}(h_l) = p_{j_l}(-h_l)$ (Parity invariance). Therefore the number of independent Falk-Peskin probabilities is equal $j_l - \frac{1}{2}$ for mesons and j_l for baryons.

Now we would like to demonstrate how Eq.(1) works by studying several examples. For simplicity, we will assume the heavy quark is purely left-handed and denote $P^H_{Q,-\frac{1}{2}\to s,h}$ and $P^{H^*}_{Q,-\frac{1}{2}\to s^*,h}$ by $P^H(h)$ and $P^{H^*}(h)$ respectively.

(1) $\underline{j_l = 0}$. In this case, all the heavy quark spin transfers to the spin $\frac{1}{2}$ baryon Λ_Q. The Falk-Peskin probability is therefore trivial.

(2) $\underline{j_l^P = \frac{1}{2}^\pm}$. This implies $(s, s^*)^P = (0^-, 1^-)$ or $(0^+, 1^+)$. For the odd parity, the doublet (H, H^*) is the usual (D, D^*) or (B, B^*) multiplet. For the even parity, these are the (D_0^*, D_1') and (B_0^*, B_1') multiplets that have not been identified experimentally. The Falk-Peskin probabilities are completely fixed by parity invariance: $(p_{1/2}(-\frac{1}{2}), p_{1/2}(\frac{1}{2})) = (\frac{1}{2}, \frac{1}{2})$. Therefore the helicity fragmentation probabilities for the doublet are given by

$$\begin{pmatrix} P^{H^*}(h) \\ \\ P^H(h) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ \\ & \frac{1}{4} & \end{pmatrix} \quad , \tag{2}$$

where the helicity h runs through the values -1, 0, $+1$ across the table. One consequence of this result is that all the heavy quark spin information is lost in this case [1].

(3) $\underline{j_l = 1}$. In this case, we have $(s, s^*) = (\frac{1}{2}, \frac{3}{2})$ and the doublet (H, H^*) can be identified as (Σ_Q, Σ_Q^*) with $Q = b$ or c quark. There is one nontrivial Falk-Peskin probability w_1 cannot be determined by heavy quark symmetry: $\{p_1(h_l)\} = (\frac{1}{2}w_1, 1-w_1, \frac{1}{2}w_1)$ where h_l runs through the values -1, 0, 1 from left to right. The helicity fragmentation probabilities for the doublet can be worked out easily:

$$\begin{pmatrix} P^{H^*}(h) \\ \\ P^H(h) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}w_1 & \frac{2}{3}(1-w_1) & \frac{1}{6}w_1 & 0 \\ \\ & \frac{1}{3}(1-w_1) & \frac{1}{3}w_1 & \end{pmatrix} \quad , \tag{3}$$

where h runs through the values $-\frac{3}{2}$, $-\frac{1}{2}$, $+\frac{1}{2}$, $+\frac{3}{2}$ across the table. We note that w_1 can be expressed in terms of the helicity fragmentation probabilities of the spin $\frac{1}{2}$

state as follows:

$$w_1 = \frac{P^H(\frac{1}{2})}{P^H(\frac{1}{2}) + P^H(-\frac{1}{2})} \ . \tag{4}$$

(4) $\underline{j_l = \frac{3}{2}}$. This implies $(s, s^*) = (1, 2)$. The doublet (H, H^*) can be identified as $D^{**} = (D_1, D_2^*)$ or $B^{**} = (B_1, B_2^*)$. There is also one nontrivial Falk-Peskin probability $w_{3/2}$ cannot be determined by heavy quark symmetry:

$$p_{3/2}(h_l) = \left(\frac{1}{2}w_{3/2}, \frac{1}{2}(1 - w_{3/2}), \frac{1}{2}(1 - w_{3/2}), \frac{1}{2}w_{3/2}\right) \ , \tag{5}$$

where the helicity h_l of the light degrees of freedom runs through the values $-\frac{3}{2}$, $-\frac{1}{2}$, $+\frac{1}{2}$, $+\frac{3}{2}$ across the table. The helicity fragmentation probabilities are given by

$$\begin{pmatrix} P^{H^*}(h) \\ \\ P^H(h) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}w_{3/2} & \frac{3}{8}(1 - w_{3/2}) & \frac{1}{4}(1 - w_{3/2}) & \frac{1}{8}w_{3/2} & 0 \\ \\ & \frac{1}{8}(1 - w_{3/2}) & \frac{1}{4}(1 - w_{3/2}) & \frac{3}{8}w_{3/2} & \end{pmatrix} \ , \tag{6}$$

with the helicity h runs through the values -2, -1, 0, $+1$, $+2$ across the table. $w_{3/2}$ can therefore be expressed in terms of the helicity fragmentation probabilities of the spin 1 state as follows:

$$w_{3/2} = \frac{P^H(-1) + P^H(1) - \frac{1}{2}P^H(0)}{P^H(-1) + P^H(1) + P^H(0)} \ . \tag{7}$$

In Ref.[3], we presented a heavy quark fragmentation model that allows us to extract a perturbative result for $w_{3/2}$. The model is based on the perturbative calculation of the fragmentation functions for b quark to fragment into P-wave heavy-heavy $(b\bar{c})$ mesons [4]. Heavy quark fragmentation functions have well-defined limit as the heavy quark mass goes to infinity [3]. The Falk-Peskin probability $w_{3/2}$ can be generalized to $w_{3/2}(y)$, $w_{3/2}(t)$, and $w_{3/2}(y, t)$ that can depend on the kinematical variables y and t of the produced meson. y and t are the scaling variables corresponding to the longitudinal momentum fraction z and the transverse momentum $p_\perp$ of the mesons with respect to the fragmentation axis, respectively, according to the following relations,

$$y = \frac{1 - (1 - r)z}{zr} \ , \quad \text{and} \quad t = \frac{|\mathbf{p}_\perp|}{r(m_b + m_c)} \ , \quad \text{with} \quad r = \frac{m_c}{m_b + m_c} \ . \tag{8}$$

In the heavy quark limit $r \to 0$, simple analytic expressions for $w_{3/2}(y, t)$, $w_{3/2}(t)$, and $w_{3/2}(y)$ were derived [3]:

$$w_{3/2}(y, t) =$$

$$\frac{3y^2(y - 1)^2t^2[4y^2 + y(y + 4)t^2 + t^4]}{(t^2 + y^2)[y^4(y - 4)^2 + y^3(24 + y - 4y^2 + y^3)t^2 + y^2(17 - 2y + 2y^2)t^4 + (1 + y)^2t^6]} \ ; \tag{9}$$

$$w_{3/2}(t) = \frac{9}{80} \cdot \left(\frac{n_1 + n_2 \,\mathrm{Arctan}\,(t) + n_3 \ln{(1+t^2)}}{d_1 + d_2 \,\mathrm{Arctan}\,(t) + d_3 \ln{(1+t^2)}} \right) \; , \tag{10}$$

with $n_1 = t(630 - 2605t^2 + 231t^4)$, $n_2 = -5(126 - 735t^2 + 160t^4 + 5t^6)$, $n_3 = -1280t(1-t)(1+t)$, $d_1 = t(105 - 812t^2 + 79t^4)$, $d_2 = -3(35 - 355t^2 + 93t^4 + 3t^6)$, and $d_3 = -144t(2-3t^2)$; and

$$w_{3/2}(y) = \frac{1}{10} \cdot \frac{(y-1)^2(12 + 8y + 5y^2)}{(8 + 4y^2 + y^4)} \; . \tag{11}$$

It has been shown [3] that $w_{3/2}(t)$ and $w_{3/2}(y)$ are always less than 0.5 for all values of y and t. The original Falk-Peskin probability is given by $w_{3/2} = \frac{29}{114}$, a result first derived by Chen and Wise [5].

The above results can be applied to the fragmentation processes $c \to (D_1, D_2^*)$ and $b \to (B_1, B_2^*)$. The prediction of $w_{3/2} \leq 0.5$ in this model implies that light degrees of freedom with helicities $h_l = \pm\frac{1}{2}$ always have a larger population than the maximum helicity states of $h_l = \pm\frac{3}{2}$. This prediction supports the speculation of Falk and Peskin [1] that the angular momentum of the light degrees of freedom with $j_l = \frac{3}{2}$ prefers to align transverse to, rather than along, the fragmentation axis. This spin alignment of the heavy quark can be detected by anisotropy measurements in the decay products from the hadronic transitions between the two doublets $(1^+, 2^+)$ and $(0^-, 1^-)$ [1]. Our heavy quark fragmentation model predicts that these anisotropies vary significantly with y and t.

An upper bound of $w_{3/2} \leq 0.24$ at the 90% confidence level has been deduced from the existing experimental data for the charm system [1]. Our prediction of $w_{3/2} = \frac{29}{114} \approx 0.25$ suggests that present experiment may be close to observing a nonzero value for $w_{3/2}$. We note that the Peterson fragmentation model, widely used in the literature, contains no spin information and it is not consistent with heavy quark symmetry. It is therefore impossible to calculate $w_{3/2}$ in such a model. On the other hand, string models of fragmentation tend to give significantly larger values of $w_{3/2}$ [1] and may already have been ruled out.

We conclude that future measurements of the Falk-Peskin probability $w_{3/2}$ for the charm and bottom systems, including the dependence of $w_{3/2}$ on the scaling variables y and t, can provide valuable insights into the dynamics of heavy quark fragmentation. Finally, it is also interesting to extend our heavy quark fragmentation model to the case of baryon to get a prediction for w_1. We will leave this challenge for more ambitious readers.

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